TRINITY COLLEGE

Mathematics Methods Units 3,4
Test 4 2019

Section 1 Calculator Free

Logarithms

STUDENT'S NAME SOLUTIONS

DATE: Tuesday 2 July **TIME:** 40 minutes **MARKS:** 42

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (7 marks)

(a) Solve exactly $e^{2x}+3e^{x}-10=0$ [4]

let $y = e^x$

$y^2 + 3y - 10 = 0$

$(y+5)(y-2) = 0$

$y = -5$ $y = 2$

NO SOLN $e^x = 2$

$x = \ln 2$

(b) Solve $\log_5 x^2 - \log_5 \frac{1}{x} = 6$ [3]

$\log_5 \frac{x^2}{\frac{1}{x}} = 6$

$\log_5 x^3 = 6$

$x^3 = 5^6$

$x = 5^2$

2. (8 marks)

(a) $\frac{d}{dx}e^{x^2-x}\ln 2$ $= (2x-1)e^{x^2-x}\ln 2$ [2]

(b) $\frac{d}{dx}\frac{1}{\ln(x^2-1)}$ $= \frac{d}{dx}\left[\ln(x^2-1)\right]^{-1}$ [3]

$= -\left[\ln(x^2-1)\right]^{-2} \cdot \frac{2x}{x^2-1}$

(c) $\frac{d}{dx}\log_6 5x^e$ $= \frac{d}{dx}\frac{\ln 5x^e}{\ln 6}$ [3]

$= \frac{1}{\ln 6}\left(\frac{d}{dx}(\ln 5 + \ln x^e)\right)$

$= \frac{1}{\ln 6}\left(0 + \frac{ex^{e-1}}{x^e}\right)$

$= \frac{ex^{e-1}}{x^e \ln 6}$

$= \frac{e}{x\ln 6}$

3. (10 marks)

(a) $\int \frac{3x^4-5}{x}dx$ $= \int 3x^3 - \frac{5}{x} \, dx$ [2]

$= \frac{3x^4}{4} - 5\ln|x| + c$

(b) $\int_1^0 \frac{-2+x}{-3x^2+12x+1}dx$ $= -\frac{1}{6}\int_1^0 \frac{-6x+12}{-3x^2+12x+1} \, dx$ [4]

$= -\frac{1}{6}\left[\ln|-3x^2+12x+1|\right]_1^0$

$= -\frac{1}{6}\left(\ln 1 - \ln 10\right)$

$= \frac{\ln 10}{6}$

(c) $\int_0^{\frac{\pi}{4}} \frac{\sin 2x}{1+5\cos 2x}dx$ $= -\frac{1}{10}\int_0^{\frac{\pi}{4}} \frac{-10\sin 2x}{1+5\cos 2x} \, dx$ [4]

$= -\frac{1}{10}\left[\ln|1+5\cos 2x|\right]_0^{\frac{\pi}{4}}$

$= -\frac{1}{10}\left(\ln 1 - \ln 6\right)$

$= \frac{\ln 6}{10}$

4. (7 marks)

Given that $\log_t 3 = x$ and $\log_t 10 = y$, then determine in terms of x and y, the values of each of the following.

(a) $\log_t 30$ $= x + y$ [1]

(b) $\log_t 2.7$ $= 3x - y$ [1]

(c) $\log_t 3t^2$ $= \log_t 3 + \log_t t^2$ [2]

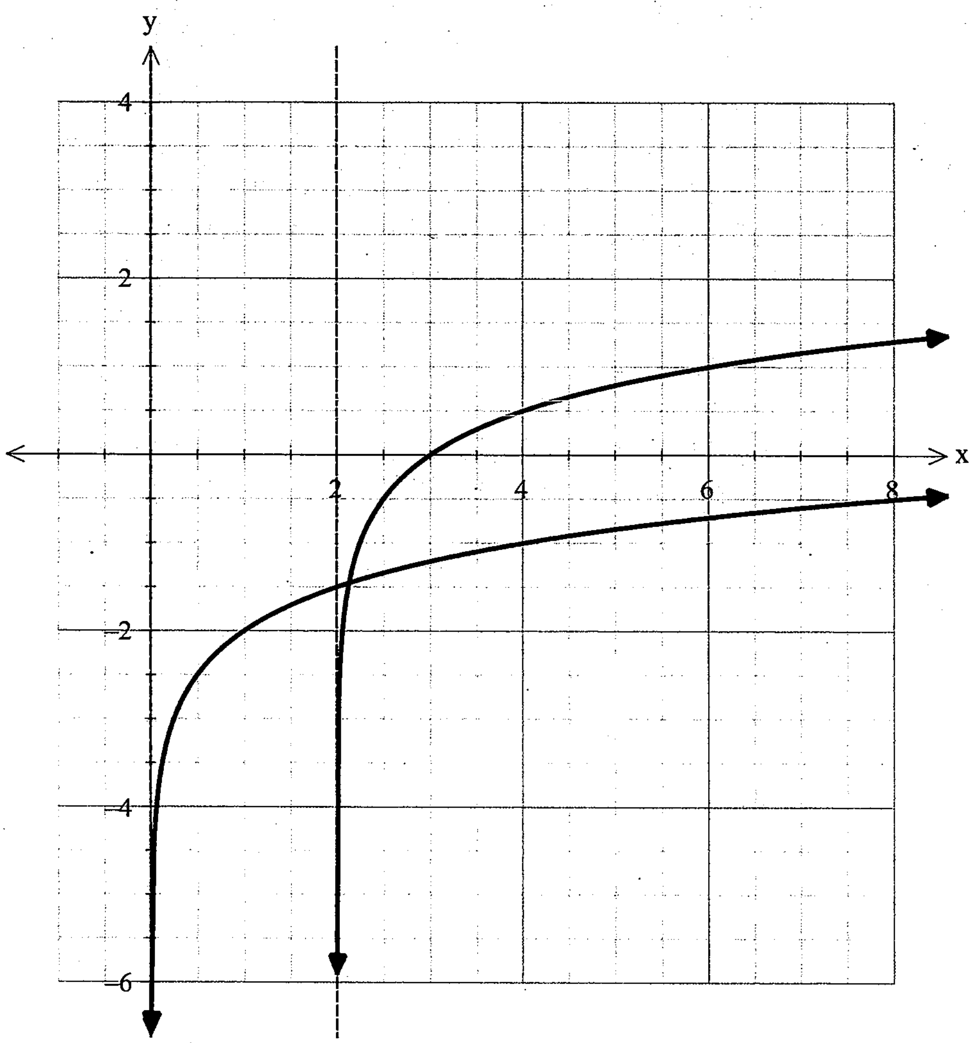
$= x + 2$

(d) $\log_t \dfrac{9t}{\sqrt{1000}}$ $= \log_t 9 + \log_t t - \frac{1}{2} \log_t 1000$ [3]

$= 2x + 1 - \frac{3y}{2}$

5. (3 marks)

The graphs shown below are for the functions $y = \log_a(x+b)$ and $y = \log_a x + c$. Determine the value a, b and c.



$a = 4$ $b = -2$ $c = -2$

6. (7 marks)

(a) Determine $\dfrac{d}{dx}x^2 \ln x$ $= 2x \ln x + x$ [2]

(b) Using the result of (a) or otherwise, determine exactly $\int_1^e (2x \ln x)dx$ [5]

$$\int_1^e \frac{d}{dx} x^2 \ln x \, dx = \int_1^e 2x \ln x \, dx + \int_1^e x \, dx$$

$$\left[x^2 \ln x\right]_1^e - \int_1^e x \, dx = \int_1^e 2x \ln x \, dx$$

$$(e^2 - 0) - \left[\frac{x^2}{2}\right]_1^e = \int_1^e 2x \ln x \, dx$$

$$e^2 - \frac{e^2}{2} + \frac{1}{2} = \int_1^e 2x \ln x \, dx$$

$$\frac{e^2}{2} + \frac{1}{2} = \int_1^e 2x \ln x \, dx$$

TRINITY COLLEGE

Mathematics Methods Units 3,4
Test 4 2019

Section 2 Calculator Assumed
Logarithms

STUDENT'S NAME ____________________

DATE: Tuesday 2 July **TIME:** 12 minutes **MARKS:** 12

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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7. (7 marks)

The displacement, x cm, of a body from the origin after t seconds can be calculated by
$x = 3t^2 - 12\ln t + 1 \qquad t > 0$

Determine

(a) the displacement and acceleration when $t = 2$ [4]

$x(2) = 13 - 12\ln 2$ cm/sec

$\dot{x} = 6t - \frac{12}{t}$

$\ddot{x} = 6 + \frac{12}{t^2}$

$\ddot{x}(2) = 9$ cm/sec^2

(b) the time when the body comes to rest [1]

$6t - \frac{12}{t} = 0$

$6t^2 - 12 = 0$

$t^2 = 2$

$t = \pm\sqrt{2}$

$t \neq -\sqrt{2}$

$t = \sqrt{2}$

(c) the distance travelled between $t = 1$ and $t = 2$ [2]

$\int_1^2 \left| 6t - \frac{12}{t} \right| dt = 3$ cm

8. (5 marks)

The approximate apparent magnitudes of two heavenly bodies are listed in the table below.

Heavenly body	Apparent magnitude m
Sirius	−1.5
Antares	1

The ratio of brightness (or intensity) $\frac{I_A}{I_B}$ of two objects A and B of apparent magnitudes m_A and m_B respectively, satisfies the equation $\ln\left(\frac{I_A}{I_B}\right) = m_B - m_A$

(a) Determine the ratio of the brightness of Sirius to Antares $\frac{I_S}{I_A}$. [2]

$\ln \frac{I_A}{I_B} = 1-(-1.5)$

$= 2.5$

$\frac{I_A}{I_B} = e^{2.5}$

(b) If the ratio of $\frac{I_{Jupiter}}{I_{Sirius}}$ is $\sqrt{e}$, determine the apparent magnitude of Jupiter. [3]

$\ln \sqrt{e} = -1.5 - m_J$

$\frac{1}{2} = -1.5 - m_J$

$m_J = -2$